Power Distribution Analysis of VLSI Interconnects Using Model Order Reduction

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***Abstract*—The analysis and simulation of effects induced by interconnects become increasingly important as the scale of process technologies steadily shrinks. While most analyses focus on the timing aspects of interconnects, power consumption is also important. In this paper, the power distribution analysis of interconnects is studied using a reduced-order model. The relation between power consumption and the poles and residues of a transfer function (either exact or approximated) is derived, and a simple yet accurate driver model is developed, allowing power consumption to be computed efficiently. Application of the proposed method to RC networks is demonstrated using a prototype tool.**

***Index Terms*—Interconnect, model order reduction, VLSI.**

I. Introduction

As the scale of process technologies steadily shrinks and the size of designs increases, interconnects have an increasing impact on the area, delay, and power consumption of circuits. Reduction in scale causes several effects: gate delays decrease due to the thinning gate oxide; interconnect resistances increase due to shrinking wire widths; the aspect ratios of interconnects have to be increased to compensate for increasing interconnect resistance; the lateral and fringing components of capacitance dominate the total capacitance of interconnects; and interconnect capacitance dominates total gate loading. These factors cause a continual increase in interconnect delays, although of course overall circuit performance continues to increase. In fact, interconnect delay is already a significant portion of the clock cycle time for large high-frequency chips [1].

In regard to power, the situation is similar in that the portion of power associated with interconnects is increasing. This is an important fact because the conventional design, analysis, and synthesis of VLSI circuits are based on the assumption that gates are the main sources of on-chip power consumption.

To verify the effects induced by interconnects, a combination of extraction and analysis is necessary. Extraction determines the capacitance and the resistance of interconnects, which can then be used to build a circuit model for the analysis of interconnect effects. For analysis (or simulation), extensive studies have been made of the use of model order reduction over the last few years, following the introduction of asymptotic waveform evaluation [2]. Model order reduction is based on approximating the Laplace-domain transfer function of a linear (or linearized) network by a relatively small number of dominant poles and zeros. Such reduced-order models can be used to predict the time-domain or frequency-domain response of the linear network.

Although there has been significant progress in the analysis and simulation of performance-related aspects of interconnects, less work has been devoted to the analysis of power consumption (or distribution) of interconnects. Furthermore, the analysis of power-related aspects of interconnects is limited to power distribution networks and deals with quantities such as IR drop, ground bounce, and electromigration.

Manuscript received September 4, 2000; revised November 19, 2001. This paper was recommended by Associate Editor K. Mayaram.

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Publisher Item Identifier S 0278-0070(02)04699-7.

In this paper, we introduce, for the first time, a method based on a reduced-order model that allows the power distribution of interconnects to be analyzed. We show that the power, which inherently involves improper integration, can be derived from the poles and residues of the transfer function, which requires only algebraic computation. When the interconnect is driven by MOSFETs and connected to the gates of MOSFETs, the load transistor can be satisfactorily approximated by a capacitor. And we show that the driver transistor can be modeled by a linear-region resistance with sufficient accuracy for power analysis.

The remainder of the paper is organized as follows. In the next section, we briefly review model order reduction techniques, especially the one based on moment matching. In Section III, a method of power distribution analysis based on a reduced-order model is introduced. In Section IV, a driver model suitable for use in the analysis of interconnect power distribution is developed. In Section V, we present results of experiments for several examples, and in Section VI we draw conclusions.

II. Model Order Reduction

Model order reduction is a technique that takes a circuit containing a large number of poles and reduces it to a smaller representation consisting of the dominant poles from the original circuit. There are two approaches to model order reduction: moment matching and matrix approximation [3]. In this section, we outline the method based on moment matching [2]. However, we stress the fact that any kind of model order reduction method can be used as part of the power distribution analysis which we present in the next section.

A lumped linear time-invariant circuit can be described by first-order differential equations

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{b}u \\ y &= \mathbf{c}^T\mathbf{x} + du \end{aligned} \tag{1}$$

where $\mathbf{x}$ is an n-dimensional state vector, $\mathbf{A}$ is an $n \times n$ matrix, u is the system's input, y is the output of interest, and d denotes the direct-coupling term. We wish to obtain the zero-state impulse response of a linear circuit described by (1), which in turn can be used to determine its response to any excitation. We apply the Laplace transform to (1) assuming zero initial conditions and ignoring the term du, which can be treated separately. Then, we obtain

$$\begin{aligned} s\mathbf{X} &= \mathbf{A}\mathbf{X} + \mathbf{b}U \\ Y &= \mathbf{c}^T\mathbf{X} \end{aligned} \tag{2}$$

where $\mathbf{X}$, U, and Y denote the Laplace transform of $\mathbf{x}$, u, and y, respectively. It follows from (2) that the transfer function, or the Laplace transform of the impulse response, defined as $H(s) = Y(s)/U(s)$, is given by

$$H(s) = \mathbf{c}^T(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{b} \tag{3}$$

where $\mathbf{I}$ is an identity matrix. If $H(s)$ has a Taylor series expansion about $s = 0$ (i.e., Maclaurin series), then it can be described by

$$H(s) = \sum_{i=0}^{\infty} m_i s^i. \tag{4}$$

Substituting (4) into (3) and equating like powers of s, it can be shown that

$$m_i = -\mathbf{c}^T\mathbf{A}^{-i-1}\mathbf{b}, \qquad i = 0, 1, \ldots. \tag{5}$$

In a reduced-order model, especially one obtained by moment matching, the transfer function is approximated by the reduced-order system of proper rational function of s having q-poles

$$\hat{H}(s) = \frac{n_{q-1}s^{q-1} + n_{q-2}s^{q-2} + \cdots + n_1 s + n_0}{s^q + d_{q-1}s^{q-1} + \cdots + d_1 s + d_0}. \quad (6)$$

Because there are $2q$ unknowns in the reduced-order system, it is forced to correspond to the first $2q$ terms of (4) by using Padé approximation, yielding the following equality:

$$\frac{n_{q-1}s^{q-1} + n_{q-2}s^{q-2} + \cdots + n_1 s + n_0}{s^q + d_{q-1}s^{q-1} + \cdots + d_1 s + d_0} = m_0 + m_1 s + \cdots + m_{2q-1}s^{2q-1}. \quad (7)$$

Multiplying both sides of (7) by the denominator of the left-hand side yields a set of equations that can be solved for $2q$ coefficients. After finding roots of the denominator of the reduced-order model, (6) can be expressed as a partial fraction expansion form given by

$$\hat{H}(s) = \sum_{i=1}^{q} \frac{r_i}{s - p_i} \quad (8)$$

where r_i is a residue of $\hat{H}(s)$ at the pole p_i. It is then straightforward to obtain the approximated impulse response $\hat{h}(t)$ from (8).

III. Analysis of Power Distribution

For a given linear or linearized circuit, the total power consumption is readily obtained. However, this does not give how the power consumption is distributed over circuit elements. In order to find the power consumption (or energy dissipation)[1] of a particular resistor element, we first obtain the reduced-order model of current flowing through the resistor, denoted by $\hat{J}(s)$ [with the corresponding time-domain function $\hat{j}(t)$], using a model order reduction techniques such as the one described in the previous section. The approximate energy dissipated by R_i, denoted by $\hat{E}_i$, during time period $[\tau_1, \tau_2]$ is then given by

$$\hat{E}_i = R_i \int_{\tau_1}^{\tau_2} \hat{j}^2(t)\, dt. \quad (9)$$

If we are interested in the total energy dissipated by a specific resistor element during signal transition, we can choose to consider a semi-infinite interval of τ, without loss of generality. We make τ_1 the time origin and τ_2 infinite time. Then $\hat{j}(t)$ will reach a steady state, provided that $\hat{j}(t)$ corresponds to the reduced-order model of an individual transition. This leads us to the improper integral

$$\hat{E}_i = R_i \int_0^{\infty} \hat{j}^2(t)\, dt. \quad (10)$$

If $\hat{J}(s)$ is obtained in the form of a partial fraction expansion such as the one in (8) and the poles are distinct, then we can readily derive an algebraic equation involving poles and residues by substituting the combination of exponentials into $\hat{j}(t)$ in (10). However, this sort of direct computation from the improper integral cannot be applied if there are poles whose orders are larger than one, or if $\hat{j}(t)$ is expressed as combination of functions other than exponentials [4]. Thus, in this paper, we resort to algebraic computation in the s plane instead of improper integration in the time domain.

First, we derive a general relation between improper integration in the time domain and algebraic computation in the s plane, which is expressed by the following proposition.

[1]*Power consumption* and *energy dissipation* are used interchangeably. More precisely, power consumption in this paper means *average power consumption*, which is equal to energy dissipation divided by the time period of interest. We restrict our focus to the charging and discharging component of the switching power thus ignoring the leakage power of active devices.

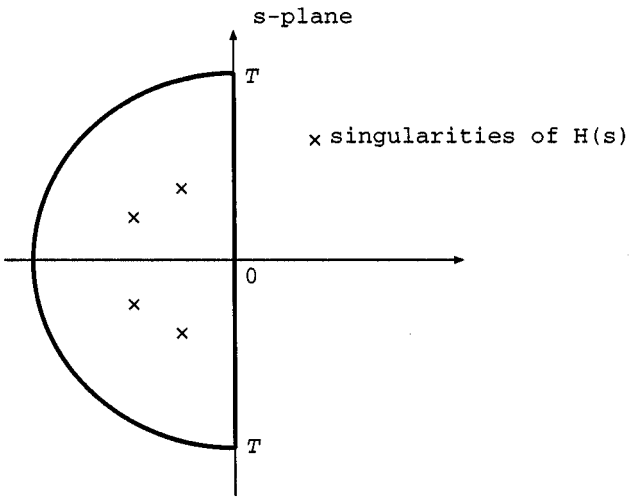

Fig. 1. The singularities of $H(s)$ and the contour of integration.

Proposition 1: If the Laplace transform of a time-domain signal $h(t)$, denoted by $H(s)$, has q singularities in the left half of the s plane, then

$$\int_0^{\infty} h^2(t)\, dt = \sum_{i=1}^{q} \tilde{r}_i \quad (11)$$

where $\tilde{r}_i$ is a residue of $H(-s)\,H(s)$ at the singularity of $H(s)$.

Proof: Let

$$I = \int_0^{\infty} h^2(t)\, dt. \quad (12)$$

From the definition of the Laplace transform, we have

$$I = \left[\int_0^{\infty} h^2(t)\, e^{-st}\, dt\right]_{s=0} = \left[\mathcal{L}\{h(t)\,h(t)\}\right]_{s=0}. \quad (13)$$

Since the Laplace transform of a product of two functions is equal to the convolution of the Laplace transforms of two functions, we find that

$$I = \left[\frac{1}{2\pi i}\lim_{T\to\infty}\int_{\gamma-iT}^{\gamma+iT} H(s-\omega)\,H(w)\, d\omega\right]_{s=0} = \frac{1}{2\pi i}\lim_{T\to\infty}\int_{\gamma-iT}^{\gamma+iT} H(-\omega)\,H(\omega)\, d\omega \quad (14)$$

where γ is chosen solely by the condition that it is to the right of the singularities of $H(s)$, meaning that γ can be chosen as any real number larger than or equal to zero. So we set $\gamma = 0$ and take the contour of integration as a semicircle of radius T with the line $\Re(s) = \gamma$ as diameter and to the left of it and the line segment $\Re(s) = \gamma,\ -T \le \Im(s) \le T$, as shown in Fig. 1. By taking T sufficiently large, we can guarantee that only the singularities of $H(s)$ fall inside the contour, because $H(-s)$ has singularities to the right of the s plane. Then, by the Cauchy residue theorem, I reduces to the sum of residues of $H(-s)\,H(s)$ at the singularities of $H(s)$. This concludes the proof. □

As an example, we consider

$$H(s) = \frac{s+3}{(s+1)^2} = \frac{2}{(s+1)^2} + \frac{1}{s+1}. \quad (15)$$

We form the product of $H(-s)$ and $H(s)$

$$H(-s)\,H(s) = \frac{(-s+3)(s+3)}{(s-1)^2(s+1)^2} = \frac{2}{(s-1)^2} - \frac{5}{2(s-1)} + \frac{2}{(s+1)^2} + \frac{5}{2(s+1)}. \quad (16)$$

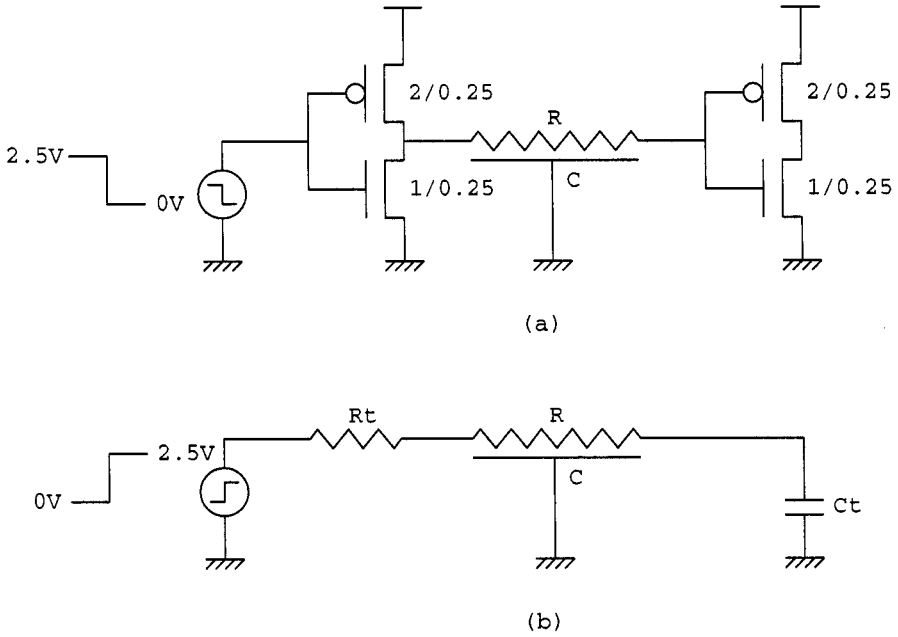

Fig. 2. A model for an interconnect driven by a MOSFET driver and loaded with a MOSFET receiver. The interconnect is 0.5-μm wide and 1-mm long in 0.25-μm technology, which results in $R = 150\ \Omega$ and $C = 0.2$ pF: (a) the circuit and (b) the model of an RC interconnect.

TABLE I
COMPARISON OF POWER CONSUMPTION, OF THE ORDER OF nW, WITH ORIGINAL AND APPROXIMATED CIRCUITS

	original circuits	approx. circuits	% error
driver	68172.0	65654.0	3.7%
R1	538.2	553.8	2.9%
R2	332.3	341.5	2.8%
R3	175.6	180.4	2.8%
R4	68.6	70.5	2.8%
R5	11.2	11.4	1.8%

It can be easily shown that

$$\int_0^\infty h^2(t)\,dt = \int_0^\infty (2te^{-t} + e^{-t})^2\,dt = \tfrac{5}{2} \tag{17}$$

which is a residue of $H(-s)\,H(s)$ at the pole $s = -1$ [the coefficient of $1/(s+1)$ in (16)]. Note that the only constraint required by Proposition 1 is that the transfer function has singularities to the left of the s plane, which is a typical situation because we are concerned mostly with stable systems. Also, note that (12) corresponds to zeroth time moment of $h^2(t)$ if its Laplace transform has a Maclaurin series expansion. If we have a reduced-order model of $H(s)$, then $\tilde{r}_i$ can be obtained by a matrix computation involving the moments of $\hat{H}(-s)\,\hat{H}(s)$ and the singularities of $\hat{H}(s)$ [2].

In the case when all the singularities are simple poles, we obtain the less complicated relation expressed by the following proposition.

Proposition 2: If the Laplace transform of a time-domain signal $h(t)$, denoted by $H(s)$, has q simple poles in the left half of s plane, then

$$\int_0^\infty h^2(t)\,dt = \sum_{i=1}^{q} r_i H(-p_i) \tag{18}$$

where r_i is a residue of $H(s)$ at the pole p_i of $H(s)$.

Proof: From Proposition 1, the residue of $H(-s)\,H(s)$ at the simple pole p_i ($\tilde{r}_i$) can be computed by

$$\tilde{r}_i = \lim_{s \to p_i} (s - p_i) H(-s)\,H(s). \tag{19}$$

Because

$$\lim_{s \to p_i} (s - p_i) H(s) = r_i \tag{20}$$

we obtain the desired result from (11), (19), (20)

$$\int_0^\infty h^2(t)\,dt = \sum_{i=1}^{q} \tilde{r}_i = \sum_{i=1}^{q} r_i H(-p_i). \tag{21}$$

□

Notice that the relations derived in Propositions 1 and 2 are exact, rather than approximate. Thus, when the reduced-order model $\hat{H}(s)$ is used in (11) or (18), the accuracy of energy dissipation is determined by the accuracies of the poles and residues of the reduced-order model. The relations can also be used to derive the exact energy dissipation if we have the Laplace transform of the exact time-domain function of current.

IV. DRIVER MODELING FOR POWER DISTRIBUTION ANALYSIS

In order to analyze power distribution based on the method outlined in Sections II and III, we need a simple linear model for nonlinear devices, such as MOSFETs, to reduce the complexity of the analysis. For the interconnect system as shown in Fig. 2(a), the drive transistor can be modeled by an equivalent resistance R_t and the load transistor by a capacitance C_t as shown in Fig. 2(b). It is well known that the receiver MOSFET can be closely approximated by a capacitor. Approximating the drive transistor by a linear-region resistance R_t is sufficiently accurate for delay estimation [5]. However, the validity of such an approximation is not obvious in the case of power distribution analysis.

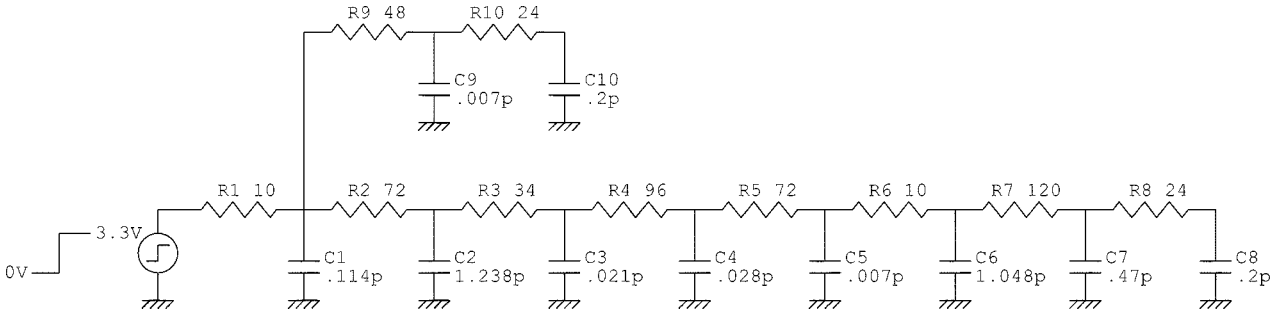

Fig. 3. An RC tree example.

The interconnect in each circuit in Fig. 2 is approximated by five sections of π-ladder circuits,[2] where R_t can be chosen as the reciprocal of the average drain conductance of the linear region or simply the reciprocal of the maximum drain conductance [5]. In order to compare the power distribution of two circuits, we simulate the power consumption of each resistor in π–ladder circuits as well as the driver itself using SPICE. The result is shown in Table I, where $R_t = 2.9$ kΩ is chosen for the approximated circuit of Fig. 2(b). The range of error in the last column clearly shows that the simple resistance approximation of the drive transistor is accurate for power distribution analysis of the interconnect systems. In order to minimize the error in the modeling, a more accurate model, which takes into account the nonlinearity of the driver characteristics and the finite input transition time [6], could be developed.

The case as shown in Fig. 2 corresponds to a fairly long interconnect driven by a buffer of intermediate size, where the buffer consumes most of power as can be seen in Table I. For an optimally (with respect to delay) buffered interconnect, however, the situation is quite different. As an example of 0.1-μm technology node parameters of ITRS [7] roadmap, the optimum length of the interconnect is about 2 mm and the optimum width of transistor channel is about 300 times that of the minimum transistor, which results in $R_t = 80\ \Omega$ and $C_t = 100$ fF. For the global interconnect line characterized by 110 Ω resistance and 140 fF capacitance, it can be shown using the theory described in this paper that 65% of the total power is consumed at the transistor and 35% is changed to heat in the interconnect.

V. Experimental Results

We implemented a prototype tool written in C^{++} based on the results presented in Sections II and III. The program reads in a circuit in a SPICE-like format and outputs the power distribution of the interconnect. Because the accuracy of power analysis depends on the accuracies of the poles and residues in the reduced-order model, the result presented in this section could be improved using more advanced techniques such as PVL [8].

A. Numerical Example

For the first example, we consider the RC tree shown in Fig. 3 [2], which is a *numerically* interesting example because of its widely varying time constants, meaning that it is difficult to approximate.[3] We compare the energy dissipation of each resistor branch obtained by SPICE with that obtained by our method (approximation by up to three poles), when a step voltage is applied. The percentage of error of the results is shown in Table II. For resistors from R3 to R8, approximation with a single pole is enough to yield an accurate result. To understand this, first note that the area under the current waveform when it is approximated by a single pole is equal to that under the exact waveform.[4] Because the exact waveform is bell-shaped (except for the driving end) while the approximated one decays monotonically, the accuracy of energy approximation with a single pole depends on the peakness exhibited by the curve, because we are interested in *the area under the square of the waveform*.

TABLE II
Comparison of the % Error of Energy Dissipation Computed Using SPICE and our Method

Resistor	1-pole	2-poles	3-poles
R1	39.1%	5.9%	3.2%
R2	14.7%	0.3%	0.4%
R3	0.9%	0.1%	0.0%
R4	0.2%	0.2%	0.0%
R5	0.2%	0.2%	0.1%
R6	0.0%	0.0%	0.0%
R7	4.2%	1.4%	0.0%
R8	7.5%	0.0%	0.0%
R9	15.3%	0.2%	0.2%
R10	13.7%	2.0%	0.0%
Avg. error	9.6%	1.0%	0.4%
Max. error	39.1%	5.9%	3.2%

As an example, in the case of waveforms for R3 (shown in Fig. 4), we would expect the squares of both waveforms to be a good match with the area under the curve. In the case of R9, on the other hand, the current waveform peaks near time zero and has a long tail along the time axis, meaning that it is highly skewed leftwards. Because a single-pole approximation based on moments gives a waveform that follows the gross shape of the original waveform ($s = 0$, thus $t = \infty$), the approximated current waveform of R9 has a large error around its peak (although the area underneath is correct) and this error becomes more significant when we compute the square of the current waveform,

[2]When $R_t = C_t = 0$, the exact energy distribution along a distributed RC interconnect can be derived, as shown in the Appendix.

[3]If R1 is scaled by ten while other resistors by 0.1, which is closer to real circuits, it is less difficult to approximate. Compared to Table II, the average and maximum error with a single pole approximation become 2.3% and 9.0%, respectively. They are 0.1% and 0.4% with two-poles approximation.

[4]If the reduced-order model of current consists of a single pole, it can be described by $\hat{I}(s) = r/(s-p)$. From (7), if we let $\hat{I}(s) = m_0 + m_1 s$ and solving for like powers of s, it can be shown that $p = m_0/m_1$ and $r = -(m_0^2/m_1)$. Thus, $\int_0^\infty \hat{i}(t)\,dt = \int_0^\infty re^{pt}\,dt = -(r/p) = m_0$. From the definition of moment, this is equal to $\int_0^\infty i(t)\,dt$.

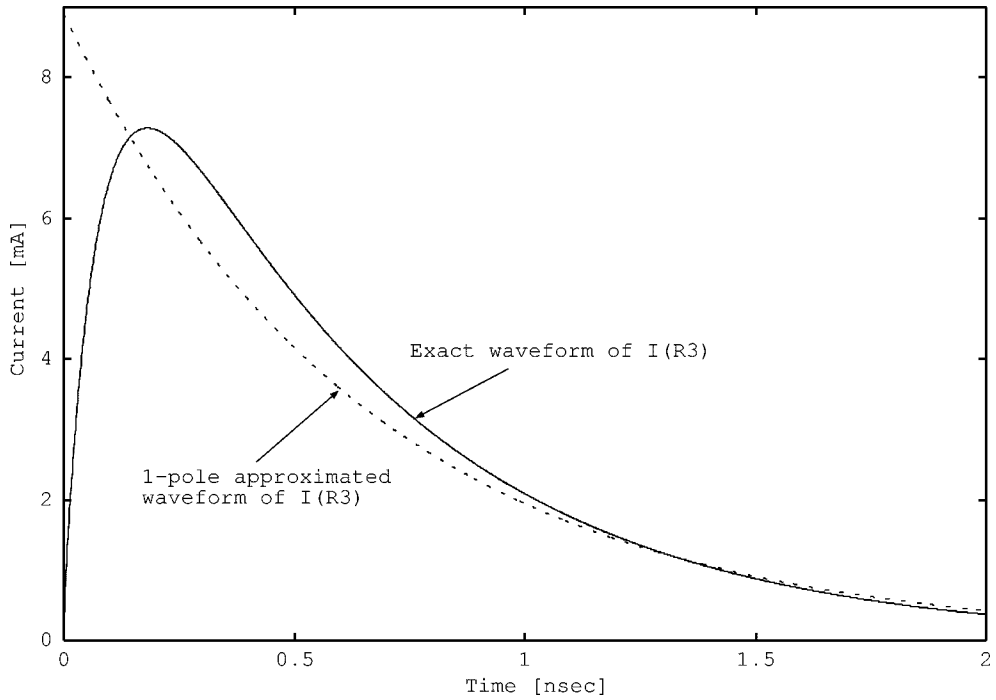

Fig. 4. Exact and approximated current waveforms at R3.

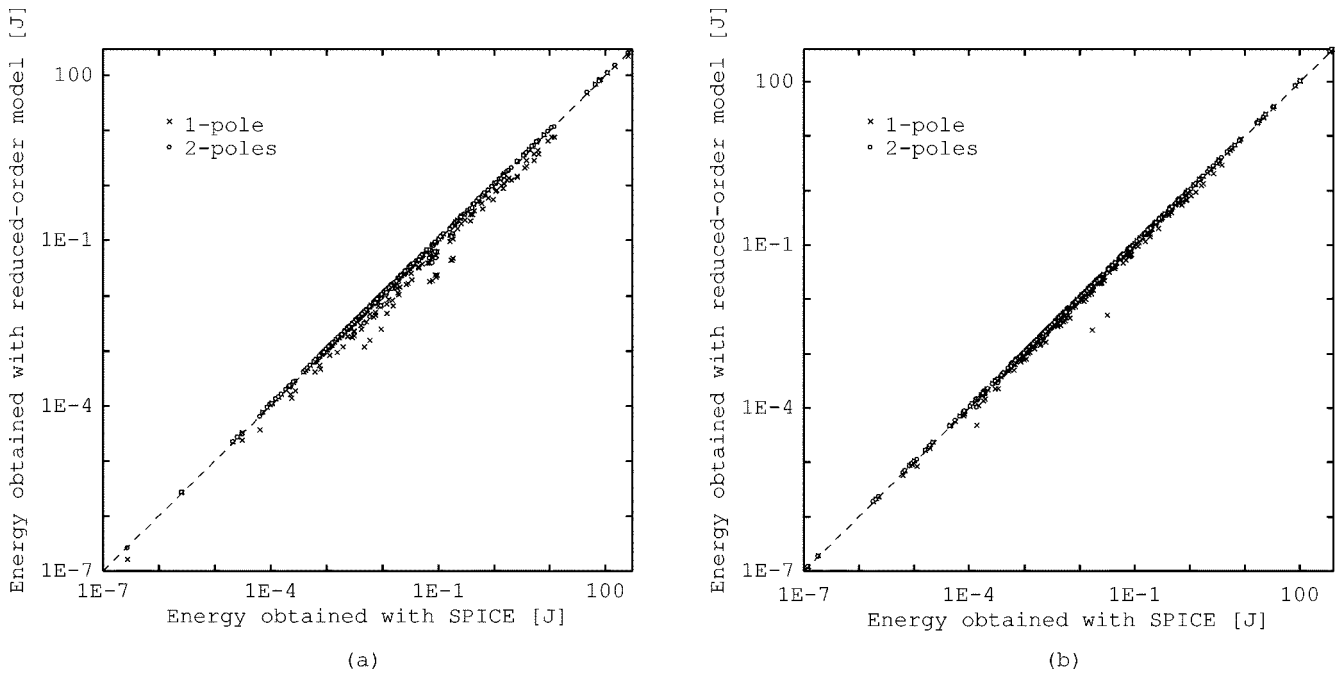

Fig. 5. Comparison of the energy distribution for randomly generated circuits: (a) circuit with 300 nodes and (b) circuit with 500 nodes.

as we must. However, approximations with more than one pole give satisfactory results for this example.

B. Random RC Networks

The second example consists of randomly generated RC tree networks.[5] We vary the number of nodes from 100 up to 1000, randomly generate resistance and capacitance values in such a way that the resulting circuit has widely varying time constants (like the first example), and compare the energy distribution obtained by SPICE with that obtained by our method. As an example, Fig. 5 shows the result for circuits with 300 and 500 nodes. Although the approximation with a single pole depends on the stiffness of the circuit, the approximation with two poles gives accurate result for most cases, as can be seen in Fig. 5.

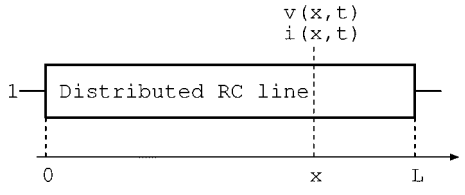

Fig. 6. A distributed RC interconnect.

[5]An RC tree is an RC network with a capacitor from each node to ground, no floating capacitors, no resistor loops, and no resistors to ground.

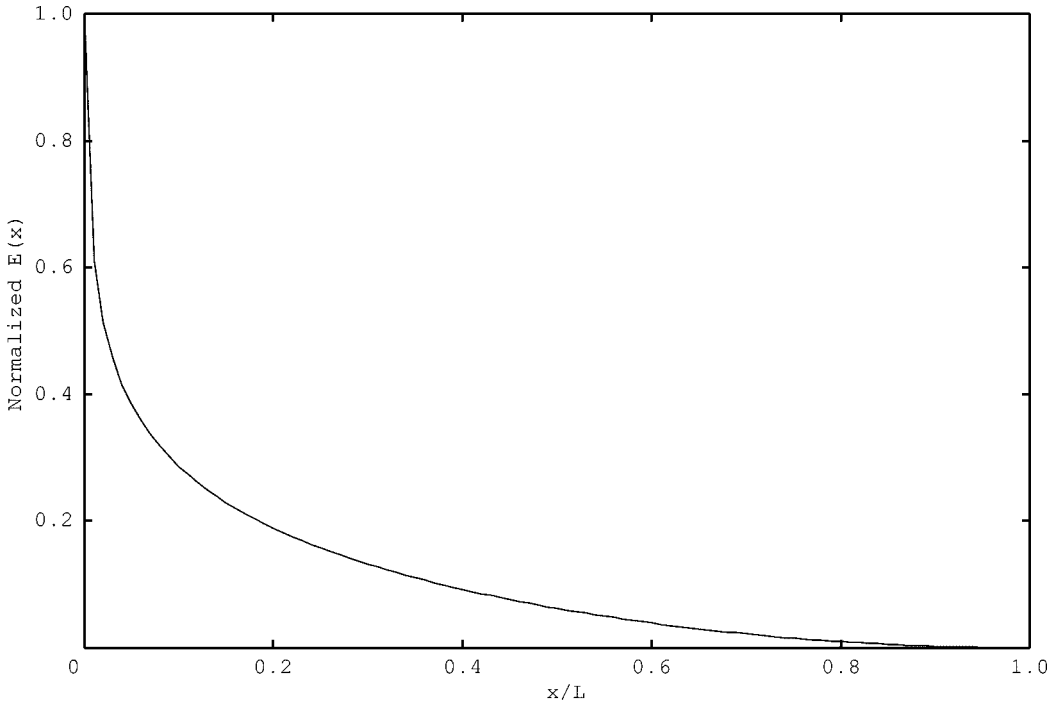

Fig. 7. The distribution of energy dissipation for a distributed RC interconnect.

VI. Conclusion

We describe a method for analyzing the power distribution of an interconnect, based on a reduced-order model. We show that power consumption can be computed efficiently in the s domain using an algebraic formulation, instead of improper integration in the time domain. The proposed method of computing power consumption relies on the poles and residues of a transfer function (whether exact or approximate) and can thus be used in any kind of model order reduction technique. We also show that MOSFETs driving an interconnect can be approximated by a linear-region resistance for power distribution analysis.

Compared to conventional delay estimation where only the receiver nodes are of interest, the computational complexity of power distribution analysis is high because we want to obtain reduced-order models of *every* resistor elements. Thus, a single-pole approximation, which guarantees enough accuracies for most of elements as shown in Table II and Fig. 5, can lead to a significant improvement in computational performance.

Appendix

Although the focus of this paper is on the power distribution analysis of circuits consisting of lumped elements, we include the exact energy distribution of a *distributed* RC interconnect for *completeness*. We consider a distributed RC line as shown in Fig. 6. Suppose that point 1 is excited by a step input. Then, the Laplace transform of $v(x,\,t)$ is given by [5]

$$V(x,\,s) = \frac{\cosh\left\{(1-x')\sqrt{sCR}\right\}}{s\cosh\sqrt{sCR}} \tag{22}$$

where $x' = x/L$ and R and C are the total resistance and capacitance of the line, respectively. Because

$$i(x,\,t) = -\frac{1}{r}\,\frac{\partial v(x,t)}{\partial x} \tag{23}$$

where r is the resistance of the line per unit length, from (22) and (23) we obtain

$$I(x,\,s) = \sqrt{\frac{C}{R}}\,\frac{\sinh\left\{(1-x')\sqrt{sCR}\right\}}{\sqrt{s}\cosh\sqrt{sCR}}. \tag{24}$$

The poles of $I(x,\,s)$ are given by

$$\begin{aligned} p_k &= -\left(k-\frac{1}{2}\right)^2\,\frac{\pi^2}{CR}, \qquad k = 1,\,2,\,\ldots \\ &= -\frac{\sigma^2}{CR}, \quad \sigma = \frac{\pi}{2},\,\frac{3\pi}{2},\,\ldots. \end{aligned} \tag{25}$$

Because all singularities are simple poles in the left half of the s plane, we can use the relation in Theorem 2. Now, from (24) and (25), we have

$$I(x,\,-p_k) = C\,\frac{\sinh\{(1-x')\,\sigma\}}{\sigma\,\cosh\sigma} \tag{26}$$

and for the residue

$$\begin{aligned} r_k &= \lim_{s\to p_k}\,(s-p_k)\,I(x,\,s) \\ &= \frac{2}{R}\,\cos\,x'\sigma. \end{aligned} \tag{27}$$

Thus, the energy dissipation at an arbitrary position x is given by

$$\begin{aligned} E(x) &= r\,\sum_{k=1}^{\infty} r_k\,I(x,\,-p_k) \\ &= r\,\sum_{\substack{\sigma=(k-1/2)\pi,\\ k=1}}^{k=\infty}\,\frac{2C}{R}\,\frac{\sinh\{(1-x')\,\sigma\}\cos\sigma\,x'}{\sigma\cosh\sigma} \end{aligned} \tag{28}$$

and it is graphically shown in Fig. 7.

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