# Electrothermal Analysis with Generalized Boundary Conditions

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*Abstract*— **Electrothermal analogy allows a resistive or RC network to be used for simple and convenient thermal analysis; however, it is only valid when heat transfer is purely convective or conductive. We investigate the analogy when a chip's operating environment is not purely convective, e.g. convective atmosphere with some source of heat flux, which is typically the practical case. We mathematically show that the analogy can be extended to such an environment, but with some modifications in a resistive network.**

**Keywords:** electrothermal equation, thermal analysis, boundary condition, thermal runaway.

## I. INTRODUCTION

Electrothermal analogy has been widely used to model heat convection as a simple resistive or RC network. A practical environment where a chip is located is often not purely convective, even though it is still approximated as convective with some calibration of parameters [1], [2].

We investigate electrothermal analogy in mathematical point of view to see how non-convective environment can still be modeled in resistive network. The electrothermal equation (ETE) is derived from fundamental heat equation for this purpose, which allows us to model boundary conditions (BCs) in generalized forms.

The new resistive network is demonstrated when environment contains constant heat flux and body radiation of a chip is taken into account. Experiments indicate that the conventional resistive network causes 5% to 6% error; more importantly, thermal runaway is sometimes undetected, while the new network always yields correct estimation.

## II. GENERALIZED BOUNDARY CONDITIONS

The fundamental heat equation is given by

$$\nabla \cdot [\kappa(\mathbf{r}) \nabla T(\mathbf{r},\ t)] = \rho C_p \frac{\partial T(\mathbf{r},\ t)}{\partial t} - p(\mathbf{r},\ t), \tag{1}$$

where $\rho$ is mass density, $C_p$ is material's specific heat, and $p$ is power density. $T$ is the temperature to be determined, and $\mathbf{r}$ is position vector. A general form of BC is given by

$$\kappa(\mathbf{r}) \frac{\partial T(\mathbf{r},\ t)}{\partial \mathbf{n}} = f(T), \tag{2}$$

where $\mathbf{n}$ is the outward normal from the surface that touches the environment and $f(T)$ represents the net heat exchange through the surface.

We take the volume integral of (1) and surface integral of (2). Their left-hand sides are equal due to divergence theorem, which allows the right-hand sides to be let equal:

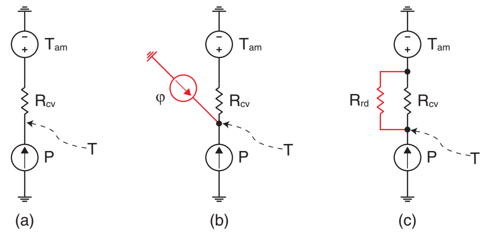

Fig. 1. Resistive network model for (a) purely convective boundary, (b) convective boundary with constant heat flux, and (c) convective boundary with body radiation.

$$V \rho C_p \frac{dT}{dt} - Vp = Af(T). \tag{3}$$

**Convective Boundary**: For purely convective boundary, $f(T)$ is given by $h(T_{am} - T)$ following Newton's law of cooling, where $h$ is the heat transfer coefficient and $T_{am} - T$ corresponds to the thermal gradient between object of interest and ambient environment. We set thermal capacitance $C_{th} = V \rho C_p$ and power consumption $P = Vp$, and note that $R_{cv} = 1/(hA)$. We now solve (3) for $T$:

$$T = T_{am} + PR_{cv} - R_{cv} C_{th} \frac{dT}{dt}. \tag{4}$$

This is a transient version of ETE. Setting $dT/dt = 0$ yields a static version:

$$T = T_{am} + PR_{cv}. \tag{5}$$

The resistive network corresponding to (5) is illustrated in Fig. 1(a).

**Convective Boundary with Constant Heat Flux**: If a BC is convective with constant heat flux (e.g. sunlight) $\phi$, $f(T)$ is given by $\phi + h(T_{am} - T)$. We solve (3) for $T$ again with new $f(T)$:

$$T = T_{am} + (P + \varphi) R_{cv} - R_{cv} C_{th} \frac{dT}{dt}, \tag{6}$$

where $\varphi = A\phi$ corresponds to the total heat flux that enters the boundary surface. Eq. (6) implies that the basic network of Fig. 1(a) can be extended if additional power source of $\varphi$ is attached in parallel to the power source of $P$ as shown in Fig. 1(b).

**Body Radiation**: Heat is transferred out through body radiation as well as through standard convection, even though body radiation is responsible for only 6%∼10% of total heat transfer [3]. The body radiation is governed by Stefan-Boltzmann law, and a BC is now given by

$$f(T) = (h_{rd} + h)(T_{am} - T). \tag{7}$$

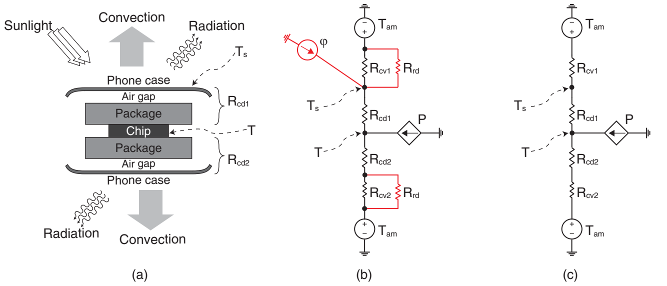

Fig. 2. (a) A smartphone under the sunlight, (b) the proposed resistive network, and (c) the conventional network.

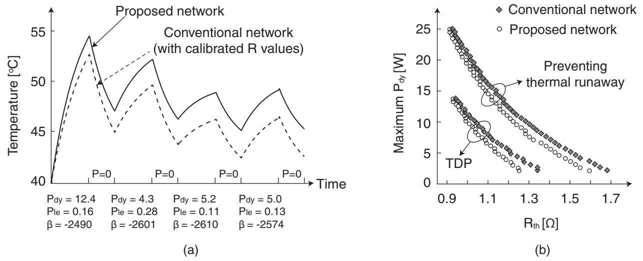

Fig. 3. (a) Transient temperature curves from our- and conventional-resistive network and (b) comparison of proposed- and conventional-network in TDP and preventing thermal runaway.

The radiative heat transfer coefficient $h_{rd}$ is defined as $\varepsilon\sigma(T_{am}^2 + T^2)(T_{am} + T)$, where $\varepsilon$ is emissivity, $\sigma$ is Stefan-Boltzmann constant [4]. Since $h_{rd}$ is a very weak function of $T$, it is usually regarded as a constant. Eq. (7) implies that body radiation can be modeled by a resistance $R_{rd} = 1/(h_{rd}A)$ placed in parallel with $R_{cv}$ as illustrated in Fig. 1(c).

## III. ASSESSMENT

We assume a smartphone under the sunlight (see Fig. 2(a)) as a test case, in which heat is convected through the atmosphere as well as radiated. Temperatures of chip and smartphone case are denoted by $T$ and $T_s$, respectively. Fig. 2(b) shows the proposed network, in which red components are newly added to take heat flux and body radiation into account, and conventional network is shown in Fig. 2(c), which serves as a reference of comparison. Chip power consumption is assumed as $P = P_{dy} + P_{le}T^2e^{\frac{\beta}{T}}$, where $P_{dy}$ is the dynamic power; the second term corresponds to power consumption due to leakage current ($P_{le}$ and $\beta$ are constants) [5].

**Steady-State Temperature**: We obtain the value of $T$ both in the proposed and conventional network while we vary chip power consumption ($P_{dy}$, $P_{le}$, and $\beta$) and ambient temperature ($T_{am}$). The conventional network underestimates $T$ by 5.8% on average (up to 8.0%). Underestimation arises because incoming heat flux is larger than outgoing heat due to body radiation, and both components are not reflected in the conventional network.

Resistance values ($R_{cv1}$, $R_{cv2}$, $R_{cd1}$, and $R_{cd2}$) of the conventional network may be arbitrarily adjusted for better accuracy, which we also try; they are varied such that sum of $\Delta T^2$ for all test cases (except for thermal runaway) is minimized. In this calibrated network, underestimation is reduced to 3.4% on average (up to 6.4%), but some thermal runaways are not correctly predicted.

**Transient Temperature**: For transient temperature analysis, we employ $C_{th}$ to be connected at a node marked with $T$ and set $T_{am}$ to 40°C. Different power consumptions are assumed at different time intervals with zero power in between representing idle or suspended state (see Fig. 3(a)).

Conventional network again underestimates the temperature by 5% on average (up to 6%). In addition to calibrating $R$ values (in the conventional network), we also adjust $C_{th}$ values so that the sum of $\Delta T^2$ at all sample time points is minimized; the new temperature curve however is almost unchanged, demonstrating the limitation of the conventional network even with empirical calibration of $R$ and $C$ values.

**Thermal Design Power (TDP)**: TDP refers to the maximum power consumption of a chip that is allowed to keep temperature below some value [6], which is set to 60°C in our experiment. We then search for maximum $P_{dy}$ for various $R_{th}$, which is aggregated resistance of $R_{cd}$ and $R_{cv}$. The results are shown in Fig. 3(b).

For the same $R_{th}$, $P_{dy}$ from proposed network is smaller by 29% on average; in other words, $P_{dy}$ from conventional network causes chip temperature to exceed 60°C and so TDP is not satisfied. For the same $P_{dy}$, $R_{th}$ from proposed network is smaller by 6.4% on average, which implies that the cooling system suggested by conventional network does not actually satisfy TDP.

Similar experiments were conducted to identify maximum $P_{dy}$ that does not cause thermal runaway. $P_{dy}$ from proposed network is smaller by 13.3% on average, which indicates that maximum $P_{dy}$ from conventional network may in fact cause thermal runaway and so is not reliable value.

## IV. SUMMARY

We have argued and also demonstrated that the basic resistive network, in which convective boundary is assumed, is not accurate, or even sometimes irrelevant (e.g. when estimating thermal runaway). We have proposed to extend the basic network to generalized boundary conditions that include external heat flux and body radiation.

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